Politecnico di Torino
1859
Dipartimento di Scienze Matematiche "G. L. Lagrange"

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Telsy

Sequential Aggregation of Multivariate Trapdoor Signatures

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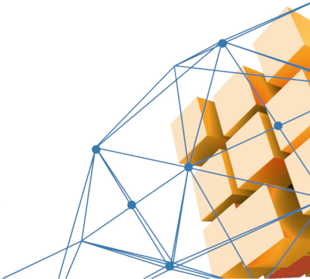
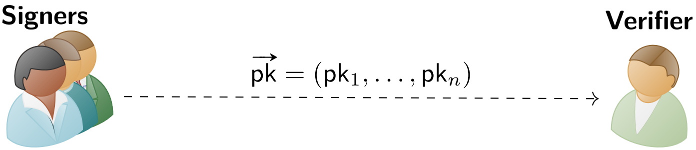
Telsy S.p.A.

Research at CryptoLabTN: Post-Quantum Cryptography

31/05/2023



Aggregate Signatures



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$$\sigma_i \leftarrow \mathsf{Sign}(\mathsf{sk}_i, m_i)$$

Aggregate Signatures

Signers

Verifier

$\overrightarrow{\mathsf{pk}} = (\mathsf{pk}_1, \ldots, \mathsf{pk}_n)$ - - - - - - - - - - →

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Combine multiple σ_i in a single Σ such that $|\Sigma| \ll \sum_i |\sigma_i|$

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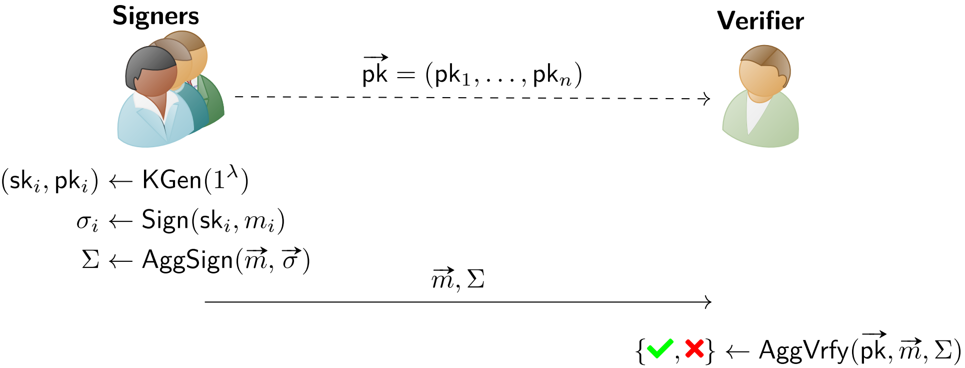

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Combine multiple σ_i in a single Σ such that $|\Sigma| \ll \sum_i |\sigma_i|$

- Reduce bandwidth consumption
- Generalize multisignatures
- Certificate chains
- Blockchains

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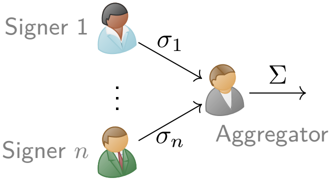
  - No interaction required by signers
  - Only known construction are based on bilinear pairings [BGLS03]



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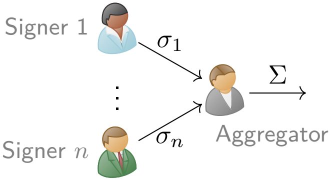

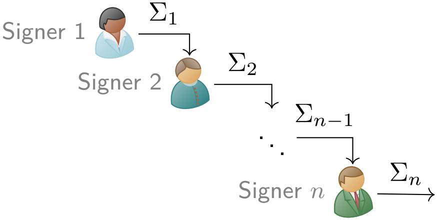

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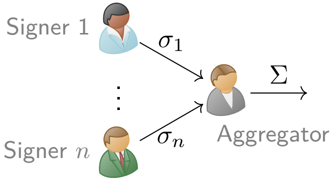

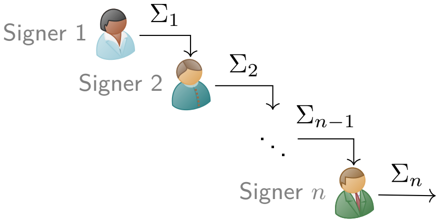

Can (S)AS be built from post-quantum assumptions?

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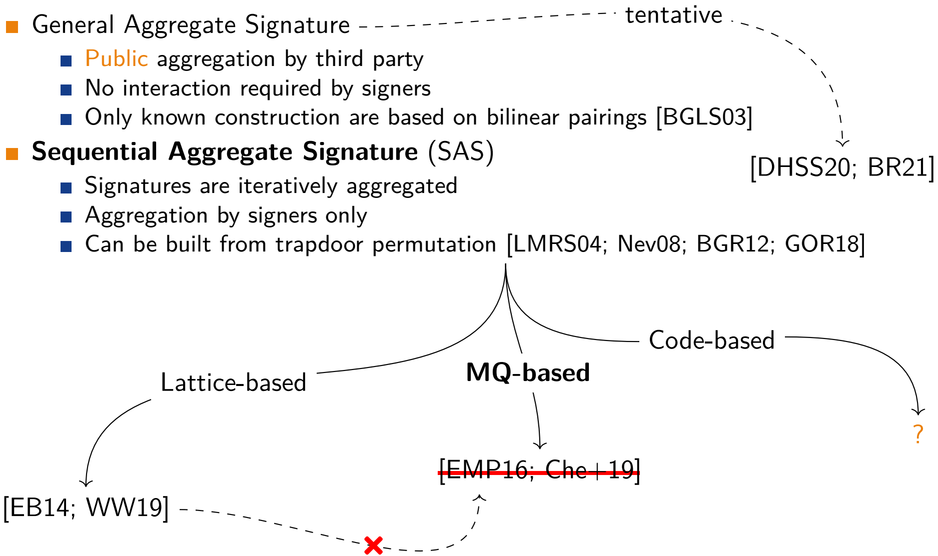

tentative

[DHSS20; BR21]

Lattice-based

MQ-based

Code-based

[EB14; WW19]

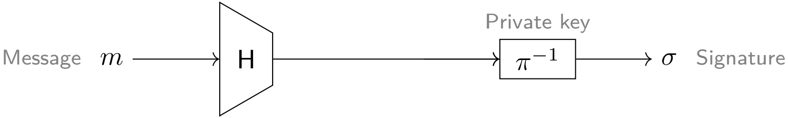
~~[EMP16; Che+19]~~

?

Can (S)AS be built from post-quantum assumptions? Yes, from lattices

SAS from trapdoor permutation

Full Domain Hash (FDH) signature from trapdoor permutation $\pi\colon \mathcal{X} \to \mathcal{X}$ and opportune hash function $\mathsf{H}\colon \{0,1\}^* \to \mathcal{X}$.



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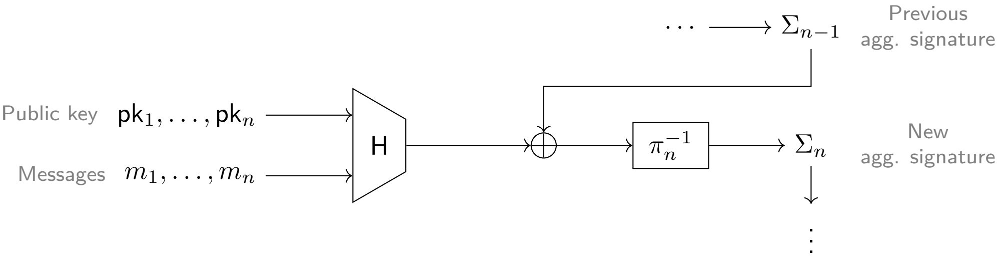

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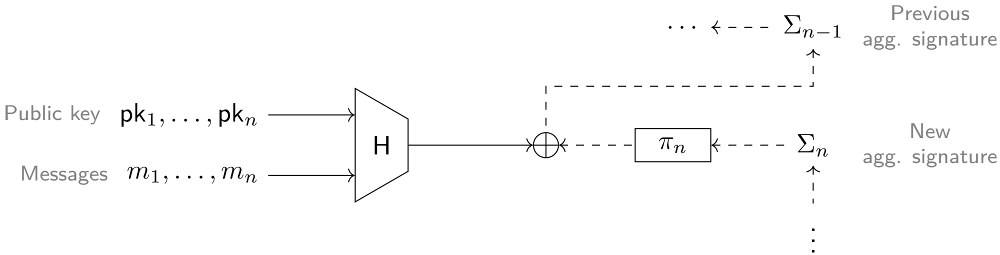

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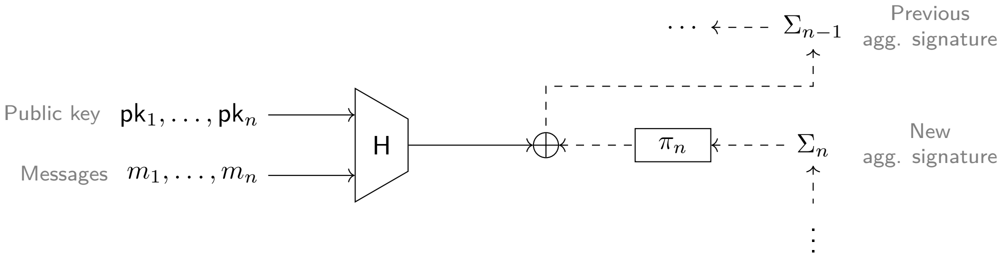

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Rigid transposition of FDH approach to post-quantum assumptions seems impractical

Trapdoor functions

A trapdoor function (TDF) T is a tuple of three algorithms $(\mathsf{TrapGen}, \mathsf{F}, \mathsf{I})$:

- $\mathsf{TrapGen}(1^\lambda)$: takes as input a security parameter 1^λ and generates an efficiently computable function $\mathsf{F}\colon \mathcal{X} \to \mathcal{Y}$ and a trapdoor I that allow to invert F.
- $\mathsf{F}(x)$: takes as input $x \in \mathcal{X}$ and outputs $\mathsf{F}(x) \in \mathcal{Y}$.
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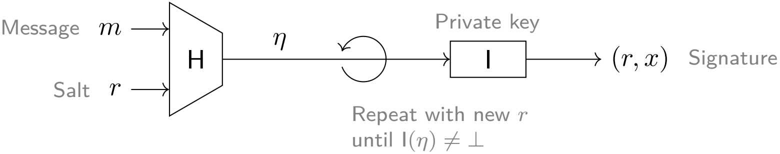
We can regain provable security using the probabilistic hash-and-sign with retry approach.

Probabilistic hash-and-sign with retry

Signature from trapdoor function (F, I) and opportune random oracle $\mathsf{H}\colon \mathcal{X} \to \mathcal{Y}$.



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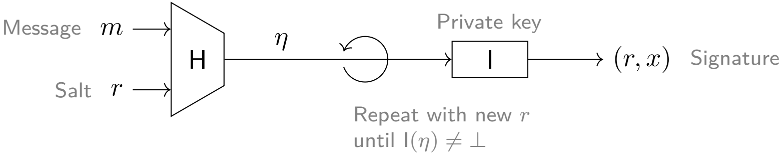


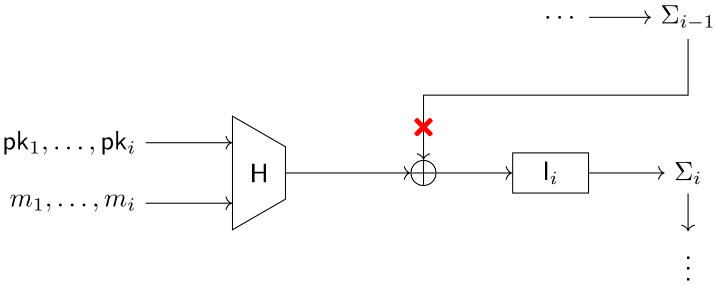
The security of the scheme is based on the one-wayness of F and the following additional property:

The output of the signing algorithm (r, x) is such that:

1. The salt r is indistinguishable from $r \leftarrow\!\!\$\ \{0,1\}^{\lambda}$.
2. The signature x is indistinguishable from $x \leftarrow\!\!\$\ \mathcal{X}$.

Generalize SAS schemes

Consider a generic trapdoor function (F, I).



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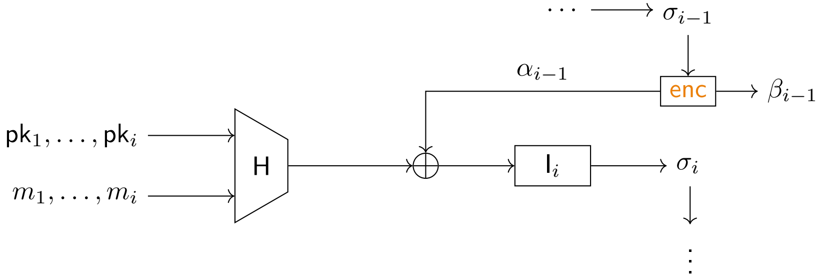

- Use an *efficient* encoding function $\mathsf{enc}\colon \mathcal{X} \to \mathcal{Y} \times \mathcal{X}'$ that splits σ_i as $\mathsf{enc}(\sigma_i) = (\alpha_i, \beta_i)$ [Nev08]

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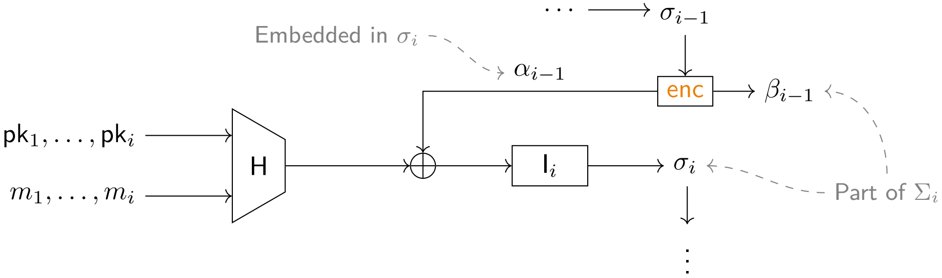

- Use an *efficient* encoding function enc: $\mathcal{X} \to \mathcal{Y} \times \mathcal{X}'$ that splits σ_i as $\mathsf{enc}(\sigma_i) = (\alpha_i, \beta_i)$ [Nev08]
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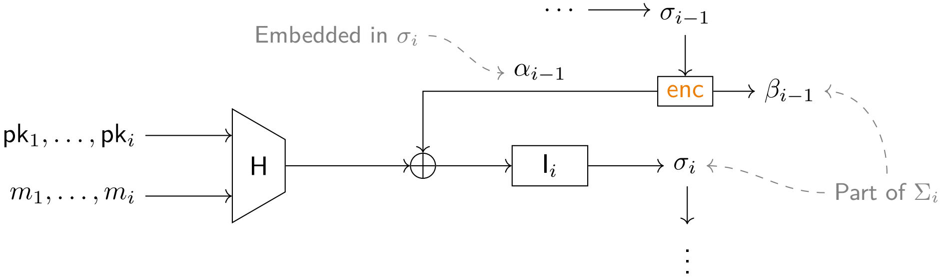

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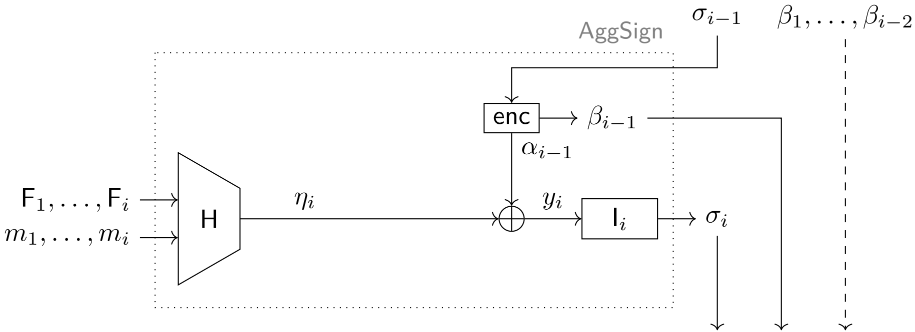
Not with UOV!

Steps to provable security

The following aggregate scheme is *not* provably secure (and sometimes provably *in*secure) with generic TDF.



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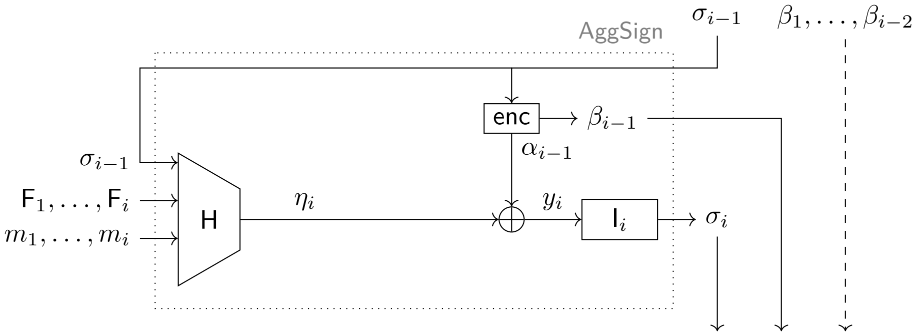

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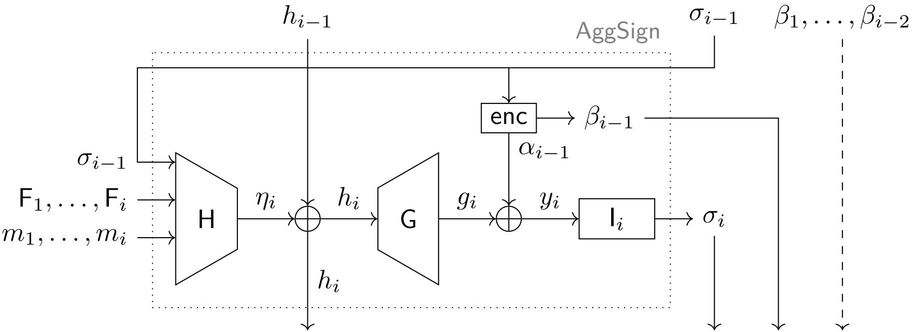

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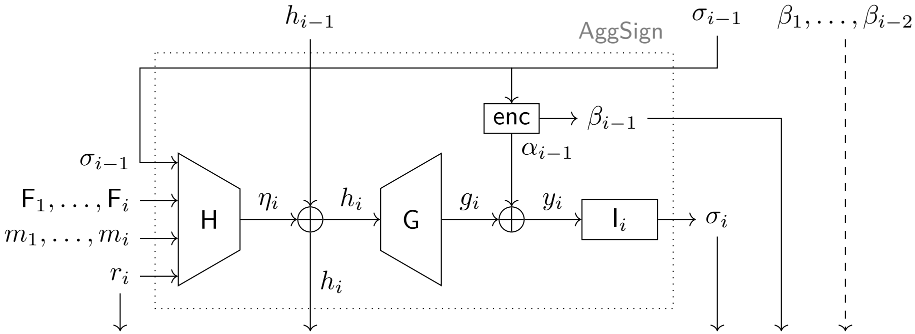
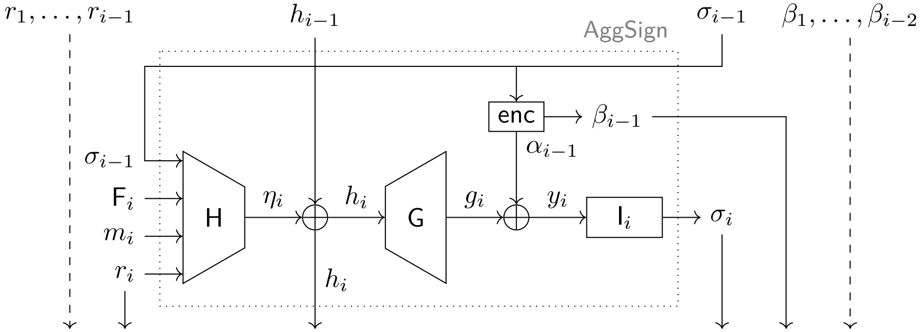


- F_i is not injective and aggregate signatures are not unique on fixed input.
- If σ_{i-1} is part of the input to H it is not possible to directly retrieve it during verification.
- Aborts on I_i may leak information.

A secure SAS scheme

The following aggregate scheme is provably secure in the random oracle model with generic TDF. Let H: $\{0,1\}^* \to \{0,1\}^{2\lambda}$, G: $\{0,1\}^{2\lambda} \to \mathcal{Y}$ be random oracles.



Compared with the previous construction

- Good: is provable secure (but not fully *black-box*)
- Good: is an **history-free** sequential aggregate signature scheme.
- Bad: the full n party signature has an overhead of length $2\lambda + n\lambda$.

Wrapping up

- Many post-quantum trapdoor signature are built from the hash-and-sign with retry approach.
- The same issues regarding provable security are also encountered for aggregated signatures.
- Inability to extend the naive FDH demonstration is the reason why simple constructions of aggregate signatures are not provable secure.

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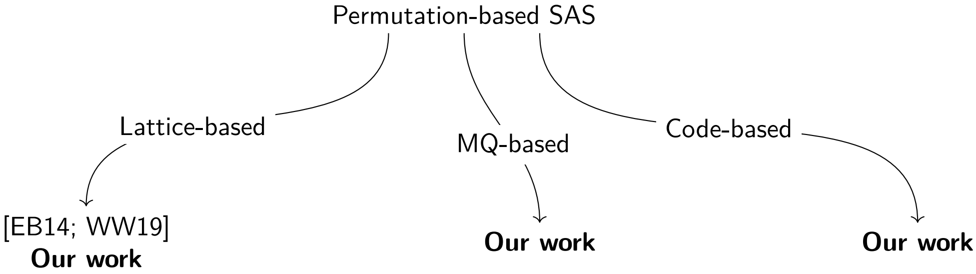

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Thank you for your attention

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