Classical Authentication in Quantum Key Distribution

Edoardo Signorini

Telsy

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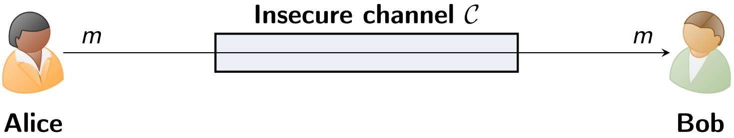
Section 1

Introduction to (ITS) authentication

Setting



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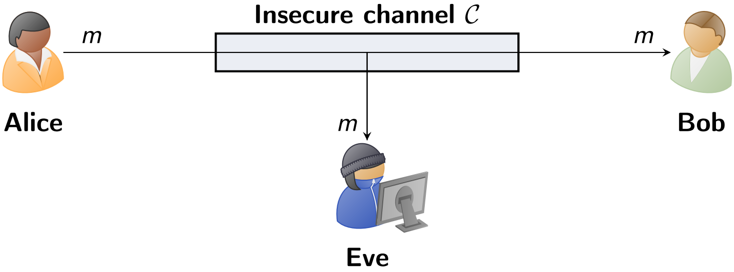

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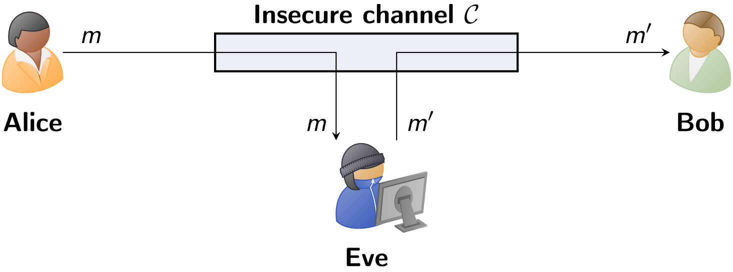

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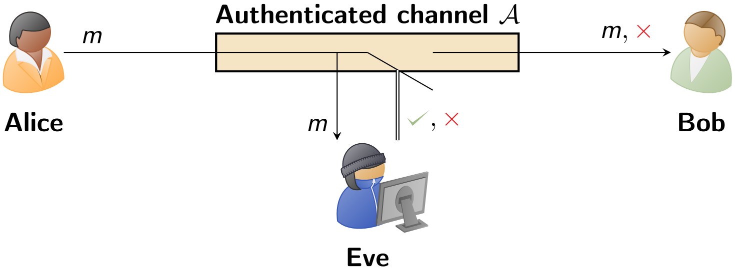

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We need an authenticated channel.

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Goal: build an authenticated channel from an insecure channel and a shared secret key.

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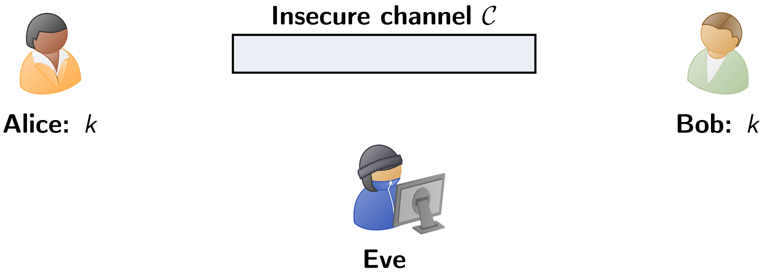

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- Choose a tag-generation algorithm $\mathrm{MAC}\colon \mathcal{K} \times \mathcal{M} \to \mathcal{T}$ and a verification algorithm $\mathrm{Vf}\colon \mathcal{K} \times \mathcal{M} \times \mathcal{T} \to \{0,1\}$.

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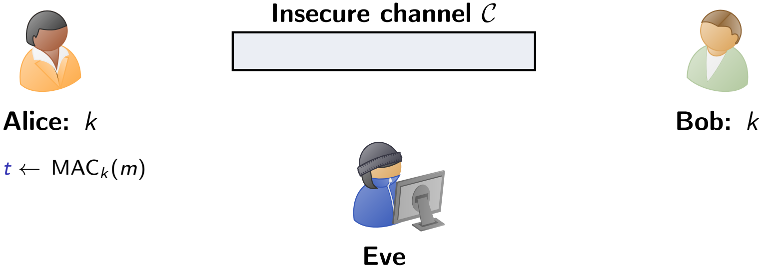

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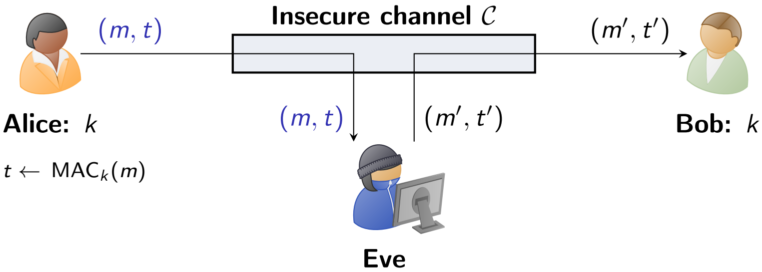

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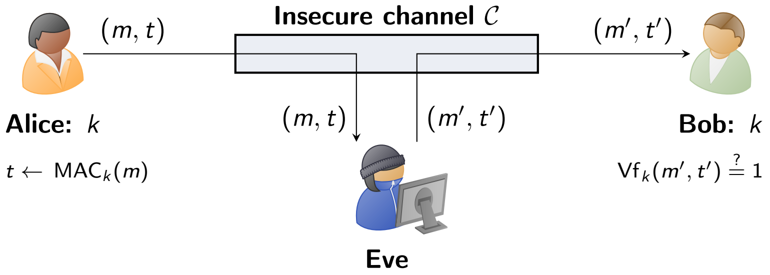

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- Bob verifies whether the received tag t' is valid.

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All the above constructions have computational security.

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A cryptographic scheme is ITS if any adversary has negligible fixed probability of breaking the scheme.

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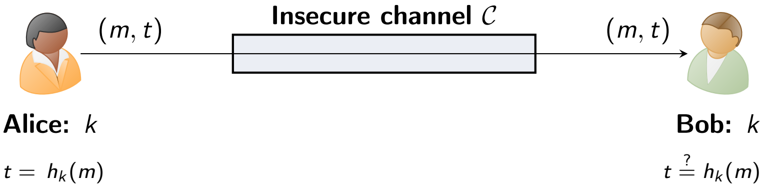
2. For any $m, m' \in \mathcal{M}$, $m \neq m'$ and $t, t' \in \mathcal{T}$

$$|\{h \in \mathcal{H} \mid h(m) = t, h(m') = t'\}| \leq \varepsilon \frac{|\mathcal{H}|}{|\mathcal{T}|}$$

One-time MAC



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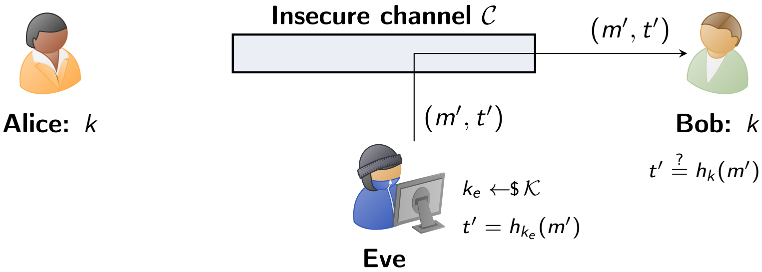

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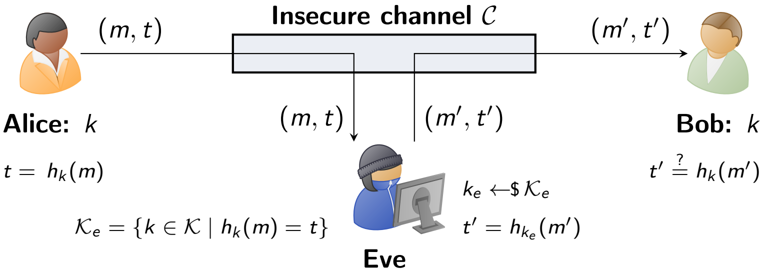

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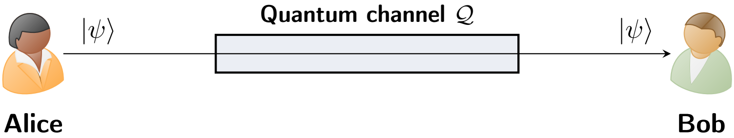
  1. Impersonate Alice, succeeding with probability $1/|\mathcal{T}|$.
  2. Substitute Alice, succeeding with probability at most ε.

Section 2

Authentication in Quantum Key Distribution

QKD in two slides I



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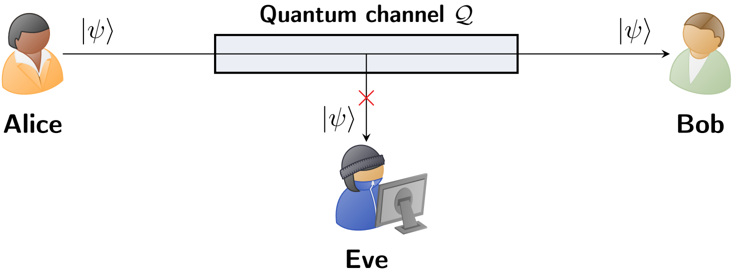

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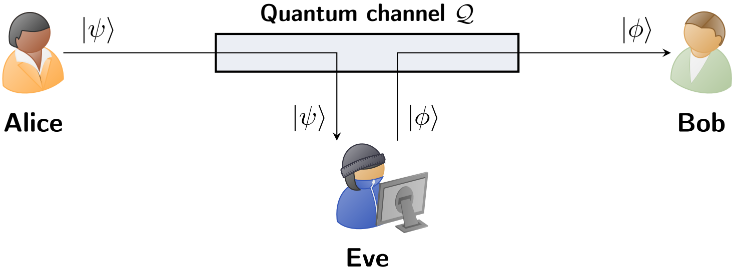

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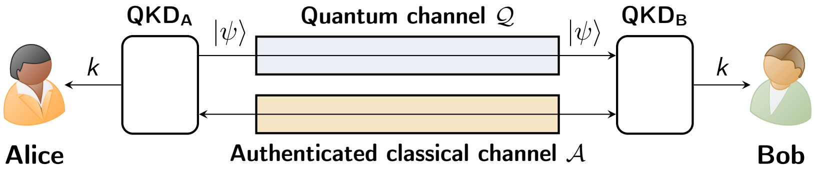
- No-cloning theorem prevents a passive attacker on the quantum channel.
- An active attacker can perform a man-in-the-middle attack.

QKD in two slides II



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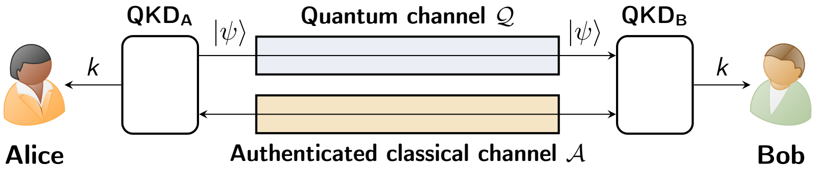

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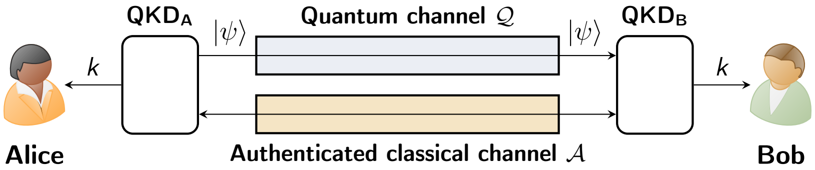

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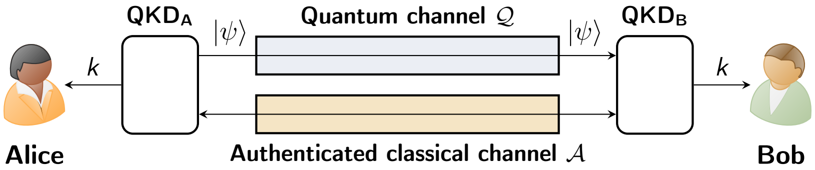

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- A portion of the exchanged key can be used as the one-time authentication key for the next round.

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Both QKD [Ben+05] and ASU_2-based one-time MACs [AL14] are proved to be secure in the Universally Composable (UC) framework.

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Idea [WC81]: recycle part of the key.

Key recycling

Let $\mathcal{T} = (\mathbb{F}_2)^t$ and let $\mathcal{H} = \{h_k : \mathcal{M} \to \mathcal{T}\}_{k \in \mathcal{K}}$ be ε-ASU$_2$. Then

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Previous results on composability do not apply directly to this scheme.

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- Non-definitive results on the authentication method based on key recycling.
- Risk of security gap between theoretical model and practical realization.

References

[AL14] Aysajan Abidin and Jan-Åke Larsson. "Direct Proof of Security of Wegman–Carter Authentication with Partially Known Key". In: *Quantum Information Processing* 13.10 (2014), pp. 2155–2170.

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Thanks for your attention

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